EXPERIMENT 3
MATCHED FILTER AND BIT-ERROR RATE (BER)

OBJECTIVES

In this experiment you will investigate the signal detection process by studying elements of a receiver and of the decoding process. In particular you will:

- investigate the characteristics of matched filters;
- study performance of various receiver structures based on different receiver filters by measuring probability of bit error;
- use the eye diagram as an investigative tool to set parameters of the detection process.

PRE-LAB ASSIGNMENT

1. A matched filter is to be designed to detect the rectangular pulse

$$r(t) = rect\Big(\frac{t - T_b/2}{T_b}\Big), \quad \text{with } T_b = 1 \text{ msec.}$$

a. Determine the impulse response of the matched filter.

b. Determine the output of the matched filter if $r(t)$ is the input.

c. Repeat parts **a** and **b** for a triangular pulse of 10 msec duration.

2. Let $Y(t) = X(t) + n(t)$, represent the waveform at the output of a channel. $X(t)$ is a polar NRZ waveform with unit pulse amplitude and binary data rate R_b of 1 kbps. $n(t)$ is a white noise process with PSD function:

$$S_n(f) = N_o/2 = 0.5 \times 10^{-4} \text{ W/Hz.}$$

If $Y(t)$ is applied to a matched-filter receiver:

a. Determine the rms value of $n(t)$ and the peak signal amplitude at the output of the matched filter.

b. Determine E_b, the average energy of $X(t)$ in a bit period.

c. Determine the probability of bit error $P_e = Q(\sqrt{2E_b/N_o})$.

3. If $Y(t)$ in Question 2 is applied to a RC-filter with frequency response:

$$H_{rc}(f) = \frac{1}{1 + j2\pi f RC},$$

with $RC = 1/(2000\pi)$,

a. determine the peak signal amplitude and rms value of the noise at the filter output;

b. determine the probability of bit error P_e, if $X(t)$ were to be detected by a receiver based on the RC-filter.

PROCEDURE

A . Characteristics of Matched Filters

A.1 Generate a rectangular pulse with unit pulse amplitude and 1 msec pulse duration.

```
≫ r = wave_gen(1,'polar_nrz',1000);
```

A.2 Display `r` and the impulse response of a matched filter based on `r`.

```
≫ subplot(311), waveplot(r)
≫ subplot(312), match('polar_nrz')
```

A.3 Observe the matched filter output if `r` is applied to its input.

```
≫ rm = match('polar_nrz',r);
≫ subplot(313), waveplot(rm)
```

> **Q3.1** Determine the time when the filter output reaches its maximum value. How is this time related to the waveform `r`?

A.4 Repeat parts A.1–A.3 for a triangular pulse with 10 msec pulse width and unit peak amplitude.

```
≫ r = wave_gen(1,'triangle',100);
≫ clf; subplot(311), waveplot(r)
≫ subplot(312), match('triangle')
≫ rm = match('triangle',r);
≫ subplot(313), waveplot(rm)
```

> **Q3.2** If the triangular pulse width is changed to 1 msec, determine the peak amplitude of the matched filter output?

A.5 Repeat parts A.1–A.3 for a manchester pulse with 10 msec pulse width and unit peak amplitude. Predict the matched filter impulse response and matched filter output. Verify your predictions using MATLAB functions.

A.6 Generate a polar NRZ waveform that represents the 5-sample binary sequence [1 0 0 1 0]. The binary data rate R_b is 1 kbps and the pulse amplitude A is 1 V.

```
≫ x5 = wave_gen([1 0 0 1 0],'polar_nrz',1000);
≫ clf, subplot(211), waveplot(x5)
```

Record the waveform `x5`

A.7 Apply `x5` to a matched filter. Record output.

```
≫ subplot(212), waveplot( match('polar_nrz',x5) )
```

> **Q3.3** Construct the waveform at a matched filter output if the input is a unipolar NRZ waveform that represents the binary sequence [1 0 0 1 0].

B . Signal Detection

B.1 Generate a 10-sample binary sequence and a waveform that represents this binary sequence in polar NRZ signalling format.

```
≫ b10 = binary(10);
≫ x10 = wave_gen(b10,'polar_nrz',1000);
≫ subplot(211), waveplot(x10)
```

B.2 Apply `x10` to a channel with 4.9 kHz bandwidth and AWGN where the noise power is 2 W. Display the channel output waveform `y10`:

```
≫ y10 = channel(x10,1,2,4900);
≫ subplot(212), waveplot(y10)
```

Decode the binary sequence from the waveform `y10`:

$\widehat{\texttt{b10}} =$

B.3 Apply `y10` to a matched filter. Display the output waveform `z10`:

```
≫ z10 = match('polar_nrz',y10);
≫ subplot(212), waveplot(z10)
```

B.4 Let T_b be the binary data period. Sample the output of the matched filter at $k\,T_b$, $k = 1, \ldots, 10$ and apply the following decision rule:

$$\widehat{b_k} = \begin{cases} 0, & \text{if sample value} > 0; \\ 1, & \text{if sample value} < 0; \end{cases}$$

where $\widehat{b_k}$ is the estimated value of the k*th* element of the binary sequence `b10`. Apply this decision rule on the matched filter output `z10`:

$\widehat{\texttt{b10}} =$

Compare your decoded sequence with the original sequence `b10`:

Q3.4 Comment on whether it is easier to decode the transmitted binary sequence directly from the channel output `y10` or from the matched filter output `z10`. If sampling instants other than those specified above are used, the probability of making a decoding error will be larger. Why?

C . Matched-Filter Receiver

C.1 Generate a 2,000-sample binary sequence `b` and a polar NRZ waveform based on `b`:

```
≫ b = binary(2000);
≫ x = wave_gen(b,'polar_nrz');
```

Apply `x` to a channel with 4.9 kHz bandwidth and channel noise power of 0.5 W. Let `y` be the channel output waveform.

```
≫ y = channel(x,1,0.5,4900);
```

C.2 Apply `y` to a matched filter. Display the eye diagram of the matched filter output `z`.

```
≫ z = match('polar_nrz',y);
≫ eye_diag(z);
```

From the eye diagram, determine the optimum sampling instants and threshold value `v_th` for the detector to decode the transmitted binary sequence `b`. Sampling instants for the matched filter output are measured with respect to the time origin. For example, if the binary data period is T_b and the `sampling_instant` parameter is set to t_i, then the detector will sample the signal at t_i, $t_i + T_b$, $t_i + 2T_b$, ... etc.

`v_th` = V.

`sampling_instant` = sec.

Use `v_th` and `sampling_instant` in the detector which will operate on the matched filter output. Record the resulting probability of bit error P_e (BER) in Table 3.1.

```
≫ detect(z,v_th,sampling_instant,b);
```

σ_n^2 $[W]$	P_e – *empirical*	P_e – *theoretical*
0.5		
1.0		
1.5		
2.0		

Table 3.1

C.3 Repeat C.1–C.2 for channel noise power of 1, 1.5, and 2 W without displaying the eye diagram of the matched filter output `z`. Record P_e results in Table 3.1. **Remark:** In Experiment 2 you have observed that the optimum sampling instants and the threshold value are independent of channel noise power. Therefore, you can use the optimum sampling instants determined in part C.2 to decode the matched filter output for different channel noise power levels.

C.4 If different sampling instants other than the optimum values are used, the resulting BER will be larger. You can observe this by decoding the binary sequence using values for the `sampling_instant` parameter that are 0.9 and 0.5 times the optimal value used in part C.3.

Q3.5 Evaluate theoretical probability of bit error values for all cases considered above and record in Table 3.1. Note that the PSD function of a white noise process can be determined as:

$$S_n(f) = \frac{N_o}{2} = \frac{\sigma_n^2}{2 \times \text{system bandwidth}},$$

where the system bandwidth in this experiment is 4.9 kHz.

D . Low-Pass Filter Receiver

D.1 Apply a rectangular pulse to a first-order RC-filter of 1 kHz bandwidth. Display the filter output and measure the peak amplitude A_r:

```
≫ r = wave_gen(1,'unipolar_nrz'); r_lpf = rc(1000,r);
≫ subplot(211); waveplot(r)
≫ subplot(212), waveplot(r_lpf);
```

$A_r =$ V.

D.2 Generate 2,000 samples from a zero-mean white noise sequence of 0.5 W power. Apply the noise sequence to the RC-filter. Record the rms value of the output noise power.

```
≫ n = gauss(0,0.5,2000);
≫ meansq(rc(1000,n))
```

$\sigma_n^2 =$ W.

> **Q3.6** From the results in parts D.1 and D.2, determine the ratio A_r/σ_n, where A_r is the peak signal amplitude measured in D.1 and σ_n is the rms value of the output noise. If `y` in part C.1 is applied to a receiver which uses the above RC-filter, determine the resulting BER.

D.3 Regenerate `y` from part C.1. Apply `y` to the RC-filter. Display the eye diagram of the output waveform `z_lpf`.

```
≫ y = channel(x,1,0.5,4900);
≫ z_lpf = rc(1000,y);
≫ clf, eye_diag(z_lpf);
```

D.4 From the eye diagram, determine the optimum sampling instant and threshold value. Decode the binary sequence form `z_lpf`.

```
≫ detect(z_lpf,v_th,sampling_instant,b);
```

Compare the resulting BER with the BER evaluated in step C.2.

D.5 Repeat part D.4 for the channel noise power of 1, 1.5, and 2 W. Record results in Table 3.2.

σ_n^2 $[W]$	P_e	
	BW = 1.0 kHz	**BW = 0.5 kHz**
0.5		
1.0		
1.5		
2.0		

Table 3.2

D.6 Repeat parts D.3 – D.5 for a first-order RC-filter with 500 Hz bandwidth. Record the resulting BER values in Table 3.2.

```
≫ z_lpf = rc(500,y);
≫ eye_diag(z_lpf)
≫ detect(z_lpf,v_th,sampling_instant,b);
```

Q3.7 Explain why the BER resulting from a low-pass filter of 500 Hz bandwidth is smaller than the BER resulting from a low-pass filter of 1 kHz bandwidth. Will the BER be further decreased if a low-pass filter of 100 Hz bandwidth is used?